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Scaling characteristics of observed oceanic rainfall in the tropics

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Motivations

- Power-law distributions have been observed for clouds and rain (e.g. Woods and Fields, 2011) which suggest emergence of self-similar structures from non-linear processes in the atmosphere.

- Recent studies even went as far to suggest that oceanic convective precipitation (Peters and Neelin, 2006) and organized mesoscale clusters (Peters et al, 2009) are critical phenomenon.

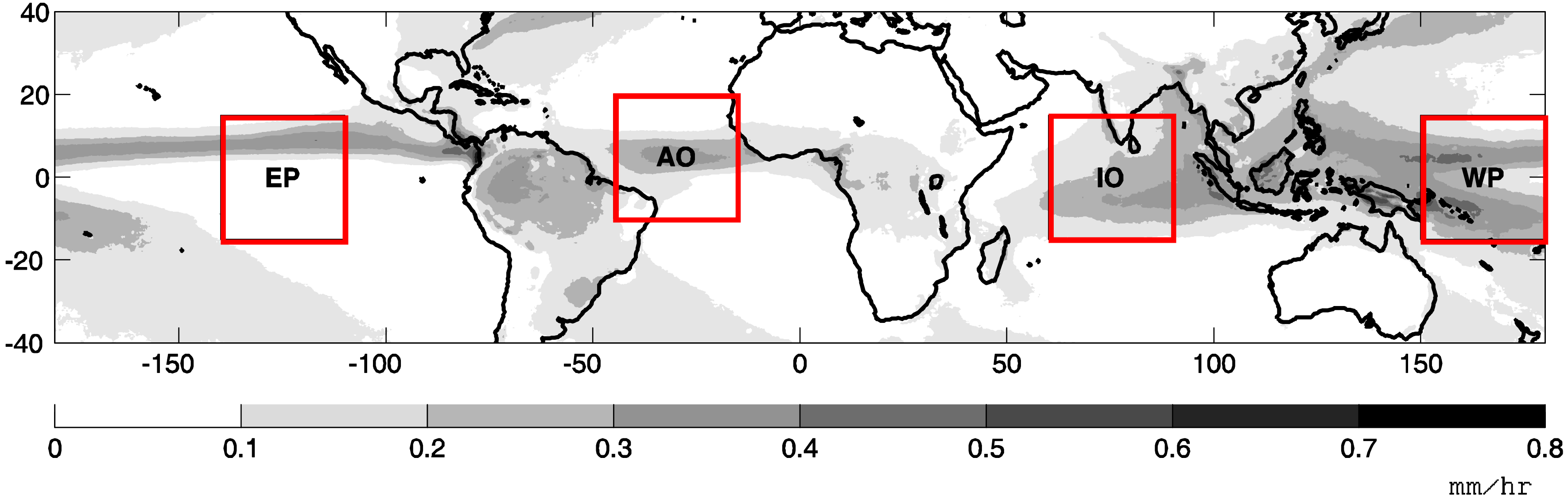
- Are tropical oceanic rain clusters self-similar objects?

    - Do the probability distributions of different observables of these clusters exhibit power-laws?

    - Are exponents same across all ocean basins?

    - Are there some relation between the exponents of different variables?

$$f(x) \propto x^{-\tau}$$


Averaged rain intensity (1998 ï 2012) from TRMM 3B42

Data used

ÅTRMM 3B42 from 1998-2012, focusing on 30°×30° domains over different ocean basins.

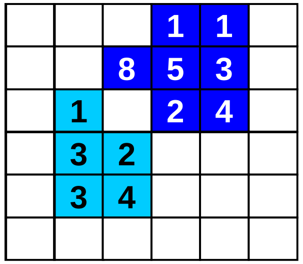
ÅResolution of the satellite data is 0.25°×0.25°.

ÅTry to avoid land masses where the scale of the precipitation systems may be imposed by topographical features (e.g. coastlines and topographies).

Method of analysis

Obtain the clusters area and cluster rain rate distributions for each of the focused area.

How clusters are identified:



			1	1	
		8	5	3	
	1		2	4	
	3	2			
	3	4			

Cluster 1: a = 5 ; r = 13

Cluster 2: a = 7 ; r = 24

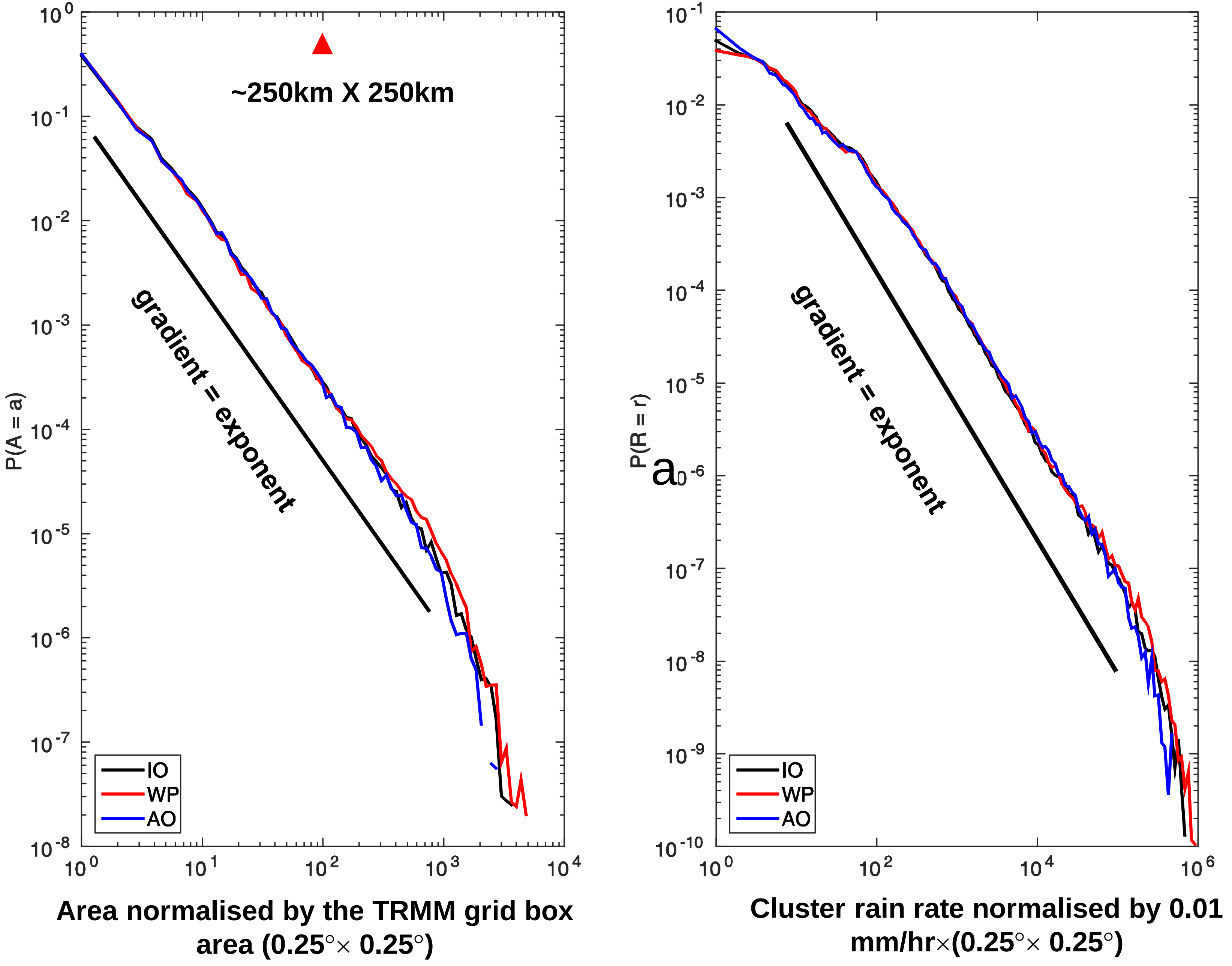
An illustration on how the cluster area (*a*) and cluster rainfall (r) are defined. Each square represents a $0.25^{o} \times 0.25^{o}$ TRMM grid box.

Note: Cluster rain rate from the data has the unit of 0.01 mm/hr $\times(0.25^{\circ} \times 0.25^{\circ})$

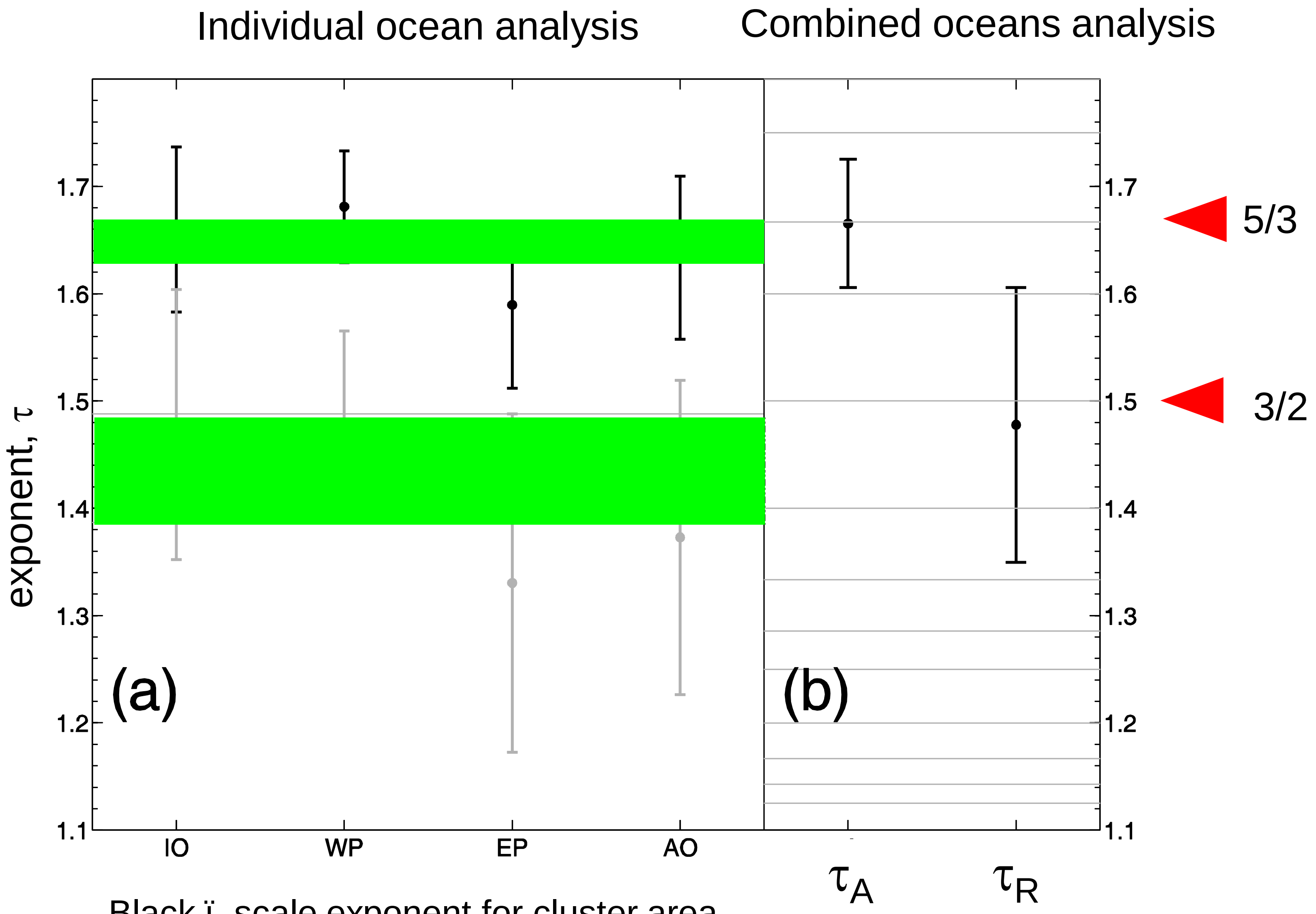
Probability of rain cluster area and rain rate

P(A=a)

P(R=r)



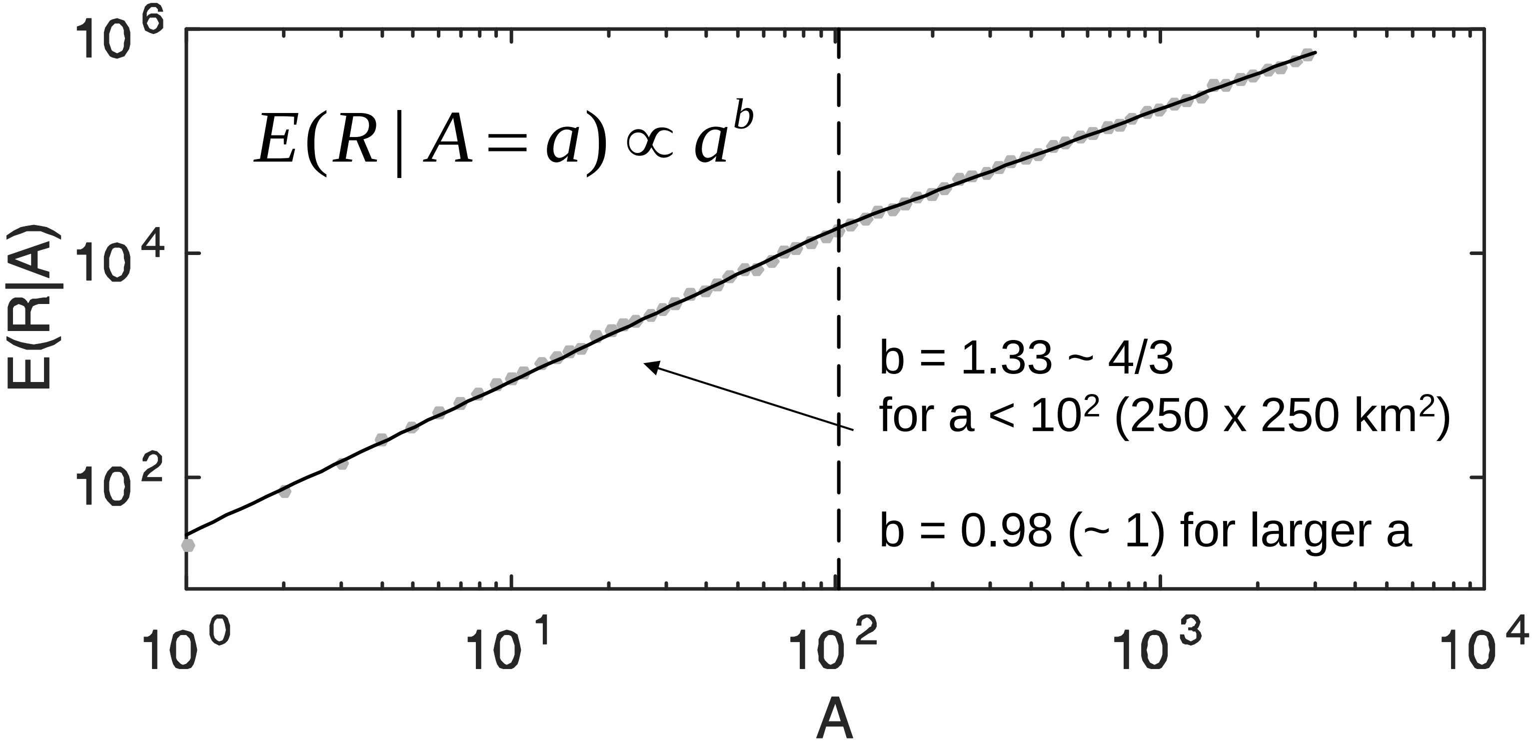
Results



Black ï scale exponent for cluster area

Grey ï scale exponent for cluster rain rate

Conditional average cluster rain rate for all oceans combined



Above give us a hint: say if $r \propto a^b$ then we have a relation for the two exponents:

$$b = \frac{\tau_A - 1}{\tau_R - 1}$$

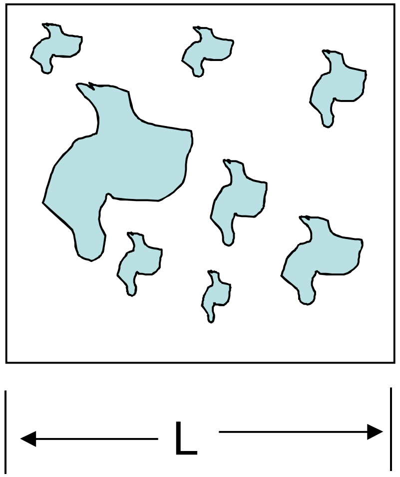
This identity is satisfied by our observed results ($b = 4/3$, $\tau_A = 5/3$ and $\tau_R = 3/2$).

What might all these mean?

(1) Finite Size Scaling Hypothesis: distributions follow simple scaling:

$$f_A(a) \propto a^{\underline{-\tau_A}} \underline{G_A(a/a_c)};\ a \ge a_0,\ a_0 \ll a_c \propto L^{\underline{D}}$$

$$f_R(r) \propto r^{-\tau_R} G_R(r/r_c);\ r \ge r_0,\ r_0 \ll r_c \propto L^{Z}$$



where $\underline{G_S(u)} \sim u^{-\tau_S},\ u \gg 1$ S = A or R

finite scaling function

$$If\ E(R\,|\,A) \propto a^{p}$$

Fundamental scaling law of narrow joint distributions:

$$p = \frac{z}{D} = \frac{\tau_A - 1}{\tau_R - 1} \Rightarrow \boxed{D(\tau_A - 1) = z(\tau_R - 1) = \Sigma}$$

Recalling our result $b = \dfrac{\tau_A - 1}{\tau_R - 1} \Rightarrow 2(\tau_A - 1) = 2b(\tau_R - 1)$

If we take $D = 2$ (reasonable to expect $a_c \propto L^2$), the cluster exponents appears to obey such fundamental law of narrow joint distributions.

(2) The exponent b is perhaps more fundamental than the scale exponents

$$r_a = E(R \mid A = a) \sim a^b$$

b = 4/3 means the cluster rain rate grows ñsuper-proportionallyò with cluster area.

This ñsuper-proportionalòrain rate is observed only up to the mesoscale (250 km x 250 km), beyond which cluster rain rate increases proportionally with cluster area.

Conclusion

Å The exponents are likely to be the identical across all ocean basins and their point estimates is close to 5/3 for cluster area and 3/2 for cluster rain rate.

Å The conditional average of cluster rain rate also scale with cluster size with an exponent of ~ 4/3.

Å It is possible that these exponents are related through a scaling relation that looks like the fundamental scaling law of narrow joint distributions.

Å The conditional mean exponent of 4/3 for mesoscale cluster suggest some positive feedback mechanisms at work within the precipitation clusters of these size.

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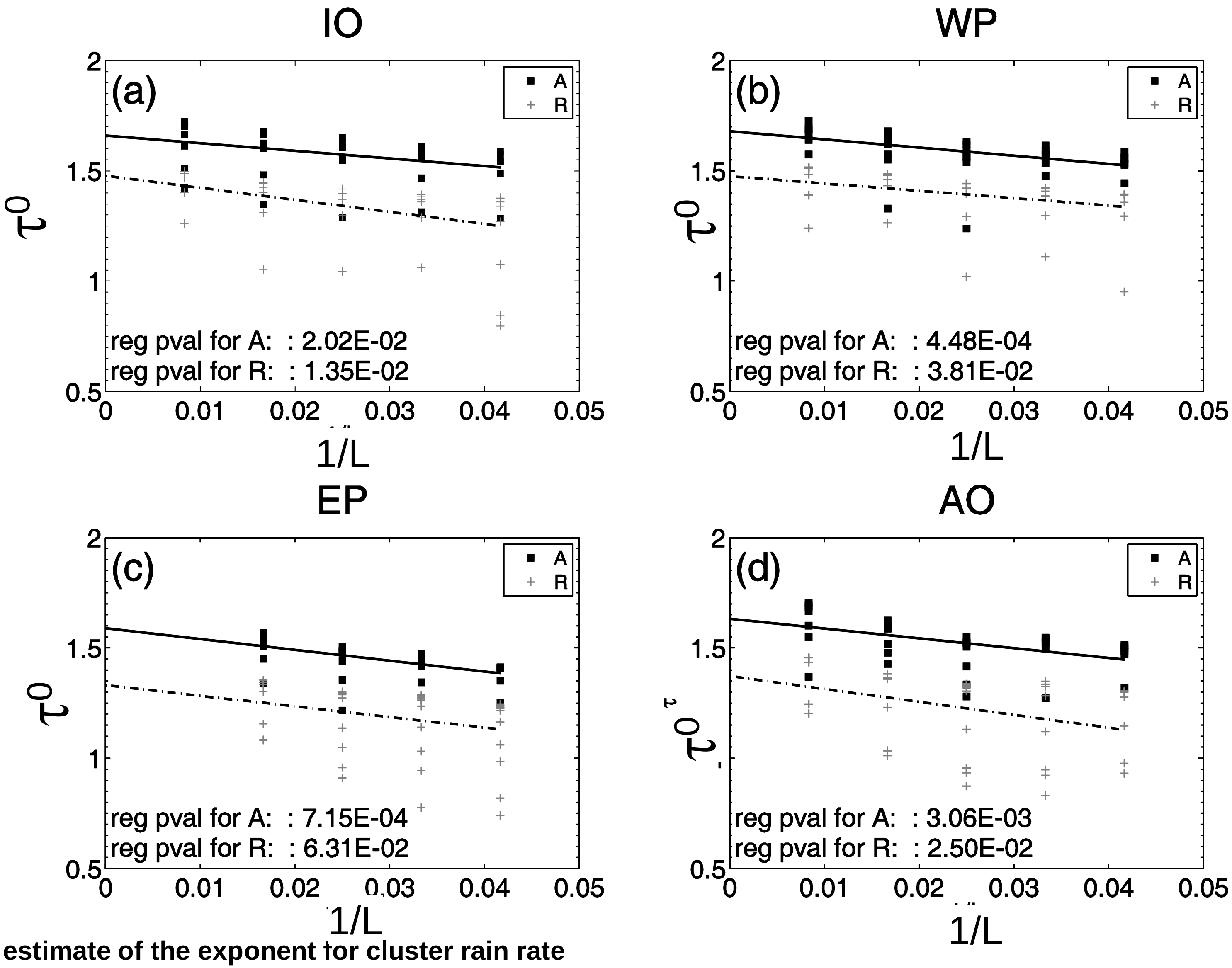
THANK YOU!

Scale exponents of cluster area and cluster rain rate



+ : one estimate of the exponent for cluster rain rate

■ : one estimate of the exponent for cluster area